# Using MUMPS as a preconditioner solver for a Hetero3D elasticity problem

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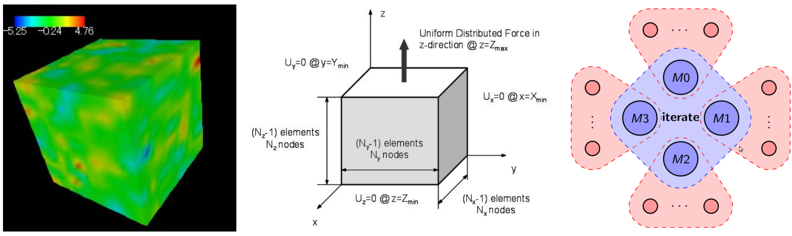
**Abstract.** This poster will present a hybrid parallel approach to solve the Hetero3D elasticity problem using direct solver MUMPS as a preconditioner.

## 1 Hetero3D

The Hetero3D solves ill-conditioned linear elasticity problems in simple cube geometries of media with heterogeneous material properties (Fig.1(a)) by parallel FEM. Tri-linear hexahedral (cubic) elements are used for the discretization, and a heterogeneous distribution of Youngs modulus in each element is calculated by a sequential Gauss algorithm, which is widely used in the area of geostatistics [1]. The boundary conditions are described in Fig.1(b). The Hetero3D code is based




(a) **distribution of Youngs modulus** (b) **boundary conditions** (c) **parallel process distribution**

**Fig. 1.** Simple cube geometries with heterogeneity as domains for 3D linear elasticity problems and parallel approach

on the framework for parallel FEM procedures of GeoFEM [2], and the GeoFEMs local data structure is applied. The Hetero3D generates parallel distributed local meshes and coefficient matrices in fully parallel manner using MPI.

⋆ This work is supported by JST-ANR FP3C Project (Collaborative Project between Japan and France on Framework).

## 2 Mumps as a preconditioner in a hybrid parallel approach

MUMPS (MUltifrontal Massively Parallel Solver) [3] is a parallel library for the solution of sparse linear equations $Ax = b$. A low-rank matrix $A$ of size $m \times n$ of numerical rank $k_\epsilon$ can be expressed with three matrices $X$ of size $m \times k_\epsilon$, $Y$ of size $k_\epsilon \times n$ and E of size $m \times n$ as $A = WZ^T + E$ , where $||E||_2 \leq \epsilon$. The parameter $\epsilon$ controls the accuracy of the approximation when ignoring $E$ and simply represent $A$ as the product of $W$ and $Z^T$; if $k_\epsilon \ll m, n$, the low rank representation allows for a considerable reduction of the memory needed to store $A$ as well as of the complexity of the linear algebra operations involving $A$.

A new low-rank format called Block Low-Rank (BLR) has been developed [4] that can be exploited within internal data structures of the multifrontal method used in the MUMPS software in order to decrease the memory consumption and the operation count of the solver.

Another interesting aspect is that low-rank approximation of the factors of a matrix A can be used as a preconditioner and we can control the quality of the approximation with $\epsilon$ versus the number of operations to generate the factors.

In order to take advantage of the potentiality of low-rank approximation, the size of the problem has to be important [4]. A first study shows that using a sequential MUMPS to perform this factorization and also the solves of the linear systems of preconditioning steps appears to be restrictive.

We will present in this poster results with a hybrid parallel approach where a parallel set of MPI processes perform the iterative solution by GPBiCG [5] and a second level of MPI processes perform the factorization and the solves with direct parallel solver MUMPS. This approach is described in Fig.1(c) where the blue processes are the processes of the iterative scheme and the set of pink ones those devoted to each direct solver instantiation.

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